Chapter 2 Logs and Exponents Review

Exponent Laws:

$x^a \cdot x^b = x^{a+b}$ $\quad x^a \div x^b = x^{a-b}$ $\quad \left(x^a\right)^b = x^{a\times b}$ $\quad x^{-a} = \left(\frac{1}{x}\right)^a$ $\quad x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Law of Logs:

$\log_a xy = \log_a x + \log_a y$ $\quad \log_a \frac{x}{y} = \log_a x - \log_a y$ $\quad \log_a x^b = b\log_a x$

$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$ $\quad \log_a \sqrt{x} = \frac{1}{2}\log_a x$ $\quad \log_{a^b} x^b = \log_a x$

Restrictions for all logs: $\log_a x$ $\quad a > 0,\ a \neq 1,\ x > 0$

Changing from logs to exponents and vice-versa: $a^y = x \longleftrightarrow \log_a x = y$

Common Exponential Equations:

Compound Interest: $A = p(1+r)^n$ $\quad r$ and n must match compound periods

Half-life: $A = A_0(0.5)^{\frac{t}{h}}$ $\quad t =$ total time $\quad h =$ half-life time

Richer Scale: $I = 10^n$ $\quad n =$ Richter scale value

Continuous growth/decay: $A = A_0 e^{kt}$ $\quad k =$ continuous growth or decay rate

Domain/Range and Asymptotes for Exponential and Log Equations:

$y = Ab^x + k,\ \ b > 0,\ b \neq 1$ $\quad D: x \in \Re$ $\quad R: y \geq k$, for $A > 0$ or $R: y \leq k$, for $A < 0$

Asymptote: $y = k$

$y = \log_a(b(x-h)) + k$ $\quad D: x \geq h$, for $b > 0$ or $D: x \leq h$, for b< 0 $\quad R: y \in \Re$

Asymptote: $x = h$

Review Questions

1. Solve without a calculator:

a) $3^{2x} = 81^{x+2}$ b) $2^{x-1} = \left(\frac{1}{32}\right)^{3x-2}$ c) $\left(\sqrt{27}\right)^{2x} = 9^{x+1}$

d) $16^{4x-8} = 64^{3-x}$ e) $\left(\frac{1}{8}\right)^{x+4} = \sqrt{32}$ f) $125^{2x-6} = \sqrt[3]{25}$

2. Express as a single logarithm:

a) $2\log a - \log b - 3\log c$ b) $\log a - 5\log b + \frac{1}{3}\log c$

c) $2\log a + 3 - \log b$ d) $\log_9 25 + \log_3 2$

e) $\frac{1}{\log_b x} - \frac{1}{\log_c x}$ f) $2\log a + 3\log a^4$

3. Simplify without a calculator:

a) $\log_2 \sqrt{32}$ b) $-6\log_2\left(\frac{1}{8}\right)$ c) $(\log_a b)(\log_b a)$

d) $\log_3\left(\sqrt{3}\right)^5$ e) $\log_7\left(7\sqrt{7}\right)$ f) $\log_4 2^6$

g) $\ln e^8$ h) $e^{\ln 3}$ i) $\frac{1}{2}\log_4 64$

4. Solve without a calculator:

a) $\log_3 x = 4$ b) $\log_5 125 - \log_2 8 = x$

c) $\log_2 x + \log_2 6 = 5$ d) $\log_3 2x - \log_3 2 = 4$

e) $\log_2[\log_x(\log_4 16)] = -2$ f) $\log 2 + \log(3x+1) = \log 7$

g) $\log_2 x + \log_2(x-6) = 4$ h) $\log_5 x + \log_5(12-x) = \log_5 27$

i) $2\log_2 x - \log_2(8-3x) = 1$ j) $2\log x - \log(24-x) = \log 2$

5. Determine the equation of the asymptotes, domain and range of:

a) $y = -2(3)^{x+1} - 4$ b) $y = 2\log(6-2x)+1$

c) the inverse of $y = 2^x - 7$ d) the inverse of $y = 4\log_3(x+2)-3$

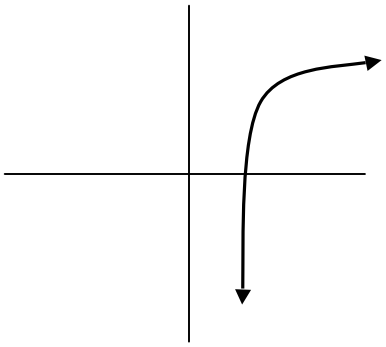
6. If $\log_3 x = 2.6$, what is the value of $\log_3 27x^{\frac{1}{2}}$?

7. If $\log_4 6 = x$ and $\log_8 3 = y$, write an expression for $\log_2 54$ in terms of x and y.

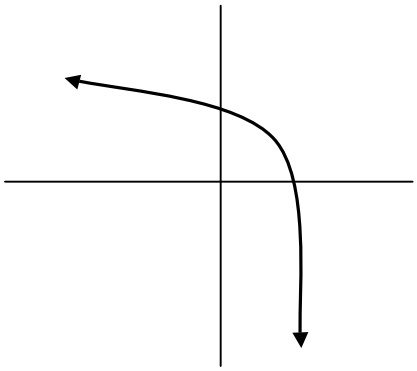
8. Given $y_1 = \log_a 6$ and $y_2 = \log_a 2$ where $0 < a < 1$, which is greater y_1 or y_2?

9. Which graph best represents the inverse of $y = -3(5)^x + 2$?

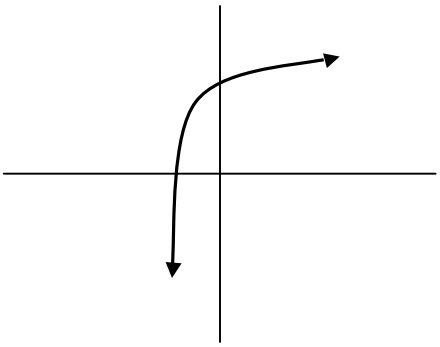
a)



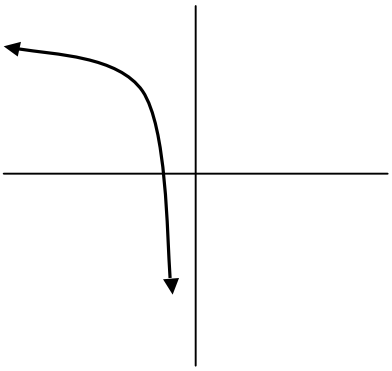
b)



c)



d)



10. Solve algebraically. Answer accurate to at least two decimal places.

a) $3(2)^{x+2} = 5^{7x}$ b) $4(5)^{x-7} = 3^{2x}$ c) $2(6)^{3-x} = 4^x$

11. Determine the equation of the inverse of $y = -\log_3(x-2)-5$.

12. ~~Graph the relation $\log_3(x-3) = 2\log_3(x+2)$.~~

13. $2500 is invested in an account that earns 6% interest per annum compounded annually. How long will it take the investment to be worth $4223.70?

14. Write an expression to determine the amount an investment of $5000 that earns 7% per annum compounded semi-annually would be worth after n years.

15. The intensity of light decreases by 4% for each meter below the surface of water in the ocean. What is the visibility at a depth of 15 meters compared to the surface of the water?

16. The half-life of a substance is 12 years. How long will it take for 14 grams of a substance to decay to 2.23 grams?

17. 330 grams of a substance decays to 46 grams in 37 hours. What is the half-life of the substance?

18. The population of a bacteria doubles every 6 hours. How long will it take 30 bacteria to grow to 200 bacteria?

19. The intensity of an earthquake in San Francisco measured 4.6 on the Richter scale. An earthquake in Texas was 210 times more intense. What was the measure of the earthquake in Texas?

20. A liquid has a pH of 3.4. Another liquid has a pH of 4.7. How many times more acidic is the first liquid?

21. The population of a city grows continuously by the formula $P = P_0 e^{kt}$ where P_0 is the initial population, t is the time in years and k is the continuous growth rate. If the population grows from 1.2 million to 2.9 million in 20 years, what is the continuous growth rate?

22. With your answer from question 21, change the formula so that it has a base of 2.

Solutions:

1. a) $x = -4$ b) $x = \frac{11}{16}$ c) $x = 2$ d) $x = \frac{25}{11}$ e) $x = -\frac{29}{6}$ f) $x = \frac{28}{9}$

2. a) $\log\left(\frac{a^2}{bc^3}\right)$ b) $\log\left(\frac{a\sqrt[3]{c}}{b^5}\right)$ c) $\log\left(\frac{1000a^2}{b}\right)$ d) $\log_3 10$
   e) $\log_x\left(\frac{b}{c}\right)$ f) $\log a^{14}$

3. a) $\frac{5}{2}$ b) 18 c) 1 d) $\frac{5}{2}$ e) $\frac{3}{2}$ f) 3
   g) 8 h) 3 i) $\frac{3}{2}$

4. a) $x = 81$ b) $x = 0$ c) $x = \frac{16}{3}$ d) $x = 81$ e) $x = 16$ f) $x = \frac{5}{6}$
   g) $x = 8$ h) $x = 3{,}9$ i) $x = 2$ j) $x = 6$

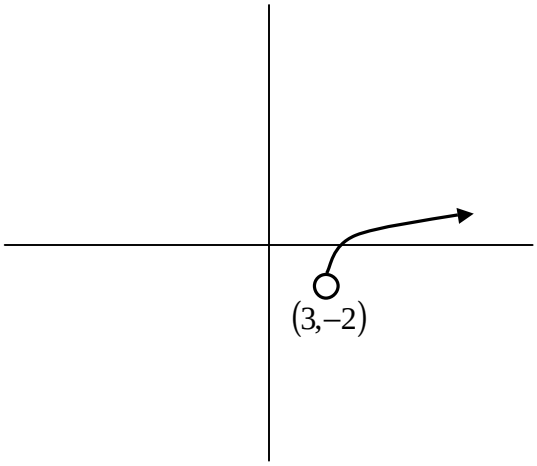
5. a) $y = -4,\ D: x \in \Re,\ R: y < -4$ b) $x = 3,\ D: x < 3,\ R: y \in \Re$
   c) $x = -7,\ D: x > -7,\ R: y \in \Re$ d) $y = -2,\ D: x \in \Re,\ R: y > -2$

6. 4.3 7. $2x + 6y$ 8. y_2 9. b

10. a) $x = 0.24$ b) $x = -16.81$ c) $x = 1.91$

11. $y = 3^{-x-5} + 2$

12.



13. 9.00 years

14. $A = 5000(1.035)^{2n}$

15. 54.21%

16. 31.81 years

17. 13.02 hours

18. 16.42 hours

19. 6.92

20. 19.95

21. 0.044

22. $P = P_0(2)^{0.064t}$